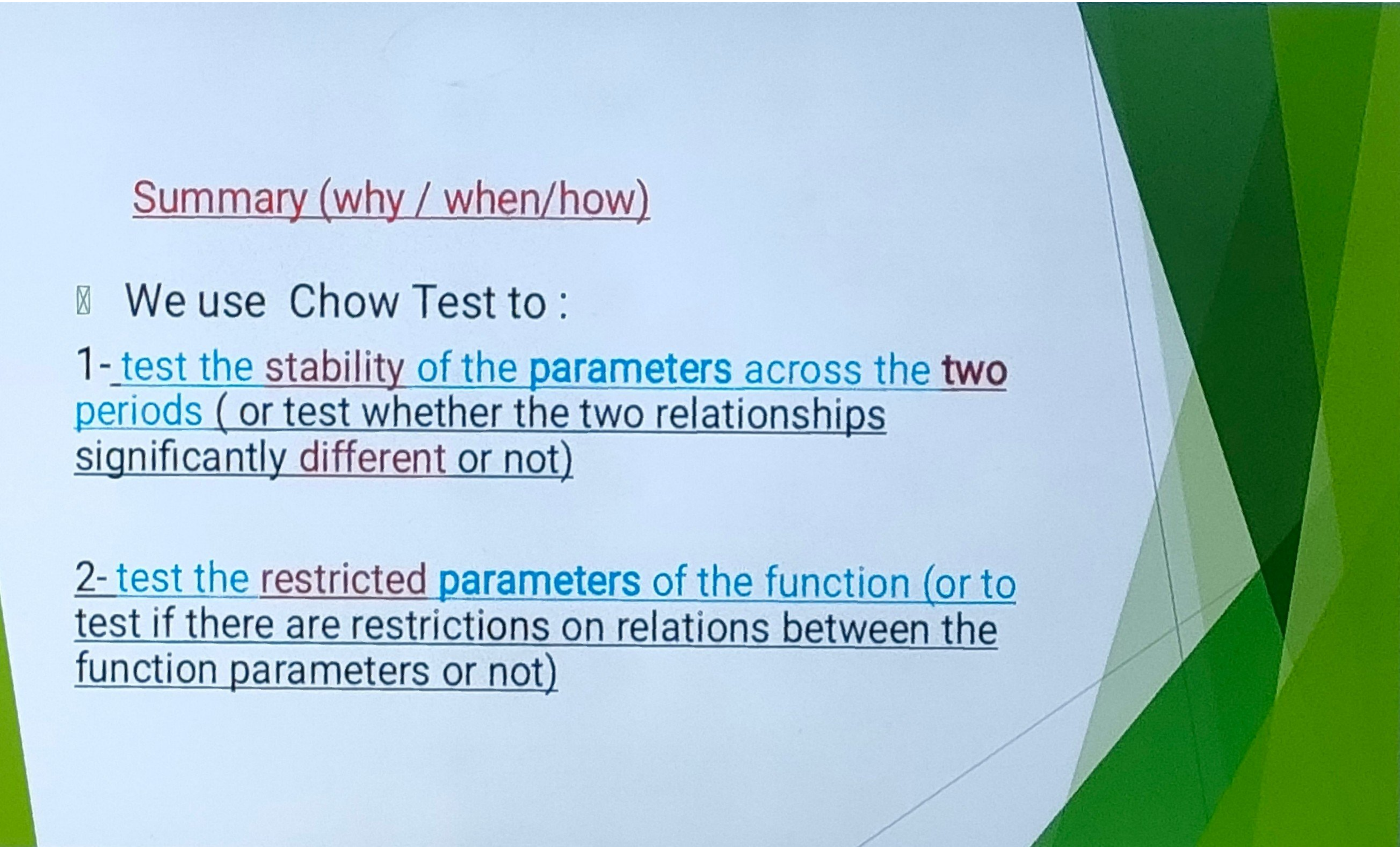
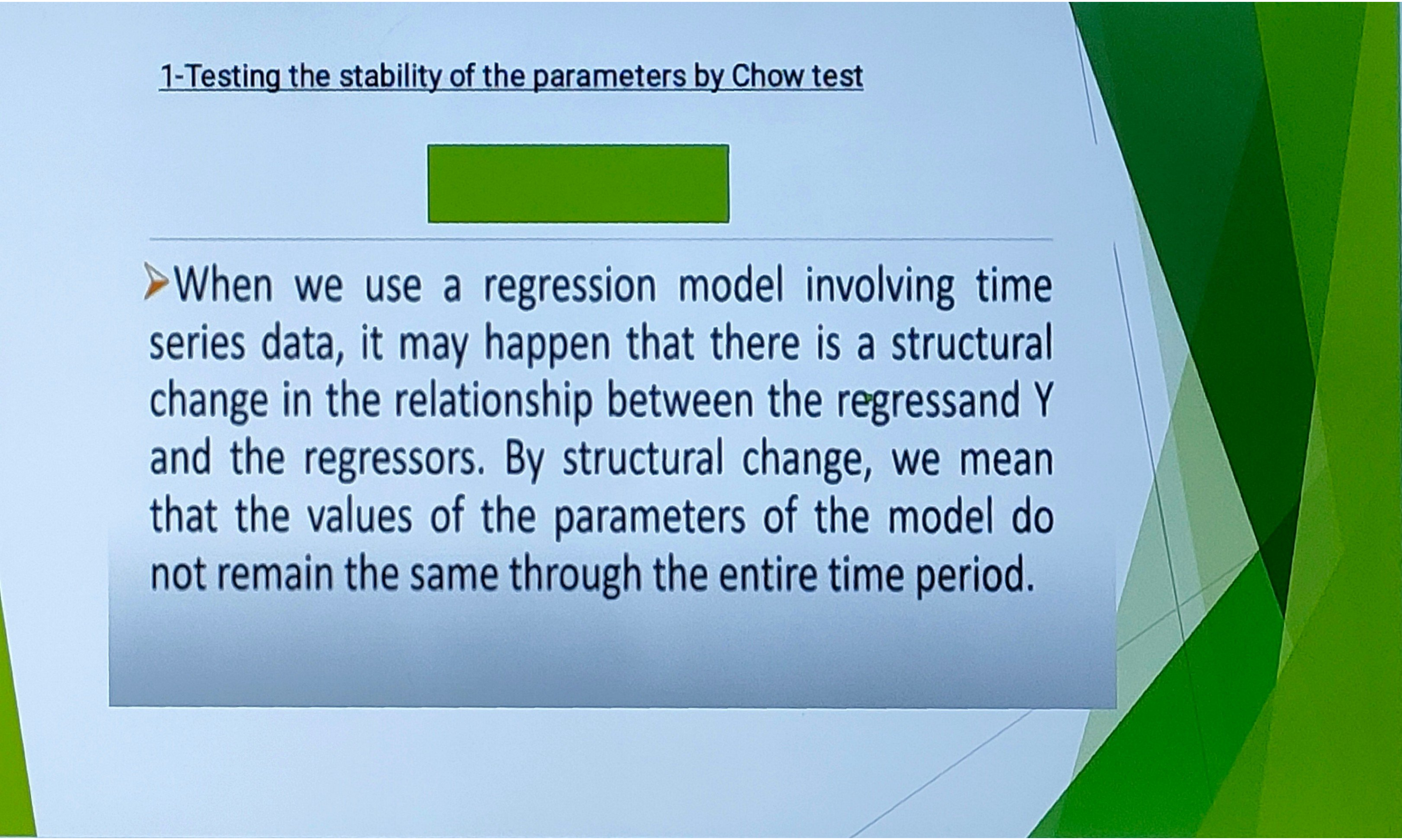


Summary (why / when/how)

- We use Chow Test to :

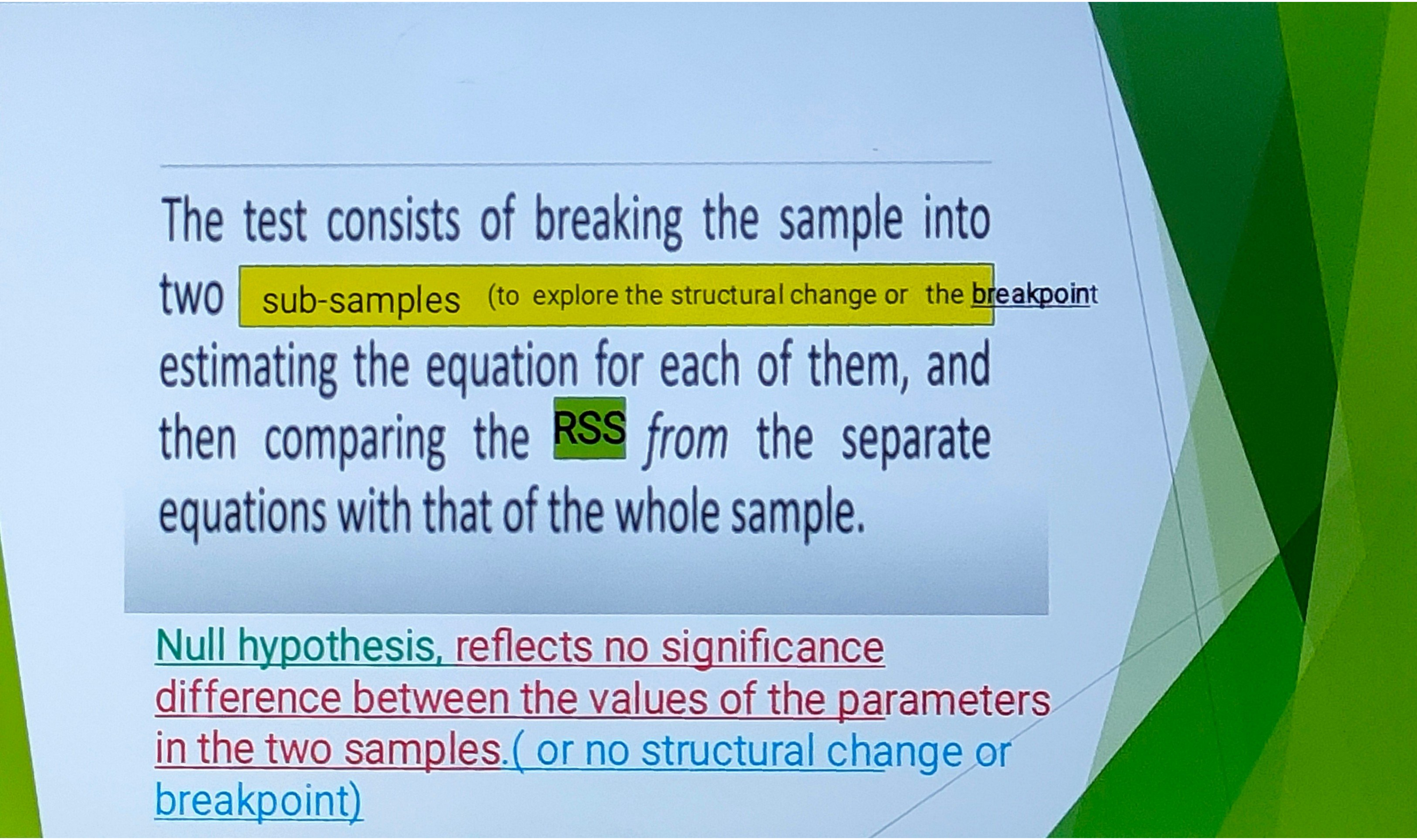
1- test the stability of the **parameters** across the **two** periods (or test whether the two relationships significantly different or not)

2- test the restricted **parameters** of the function (or to test if there are restrictions on relations between the function parameters or not)



1-Testing the stability of the parameters by Chow test

- When we use a regression model involving time series data, it may happen that there is a structural change in the relationship between the regressand Y and the regressors. By structural change, we mean that the values of the parameters of the model do not remain the same through the entire time period.



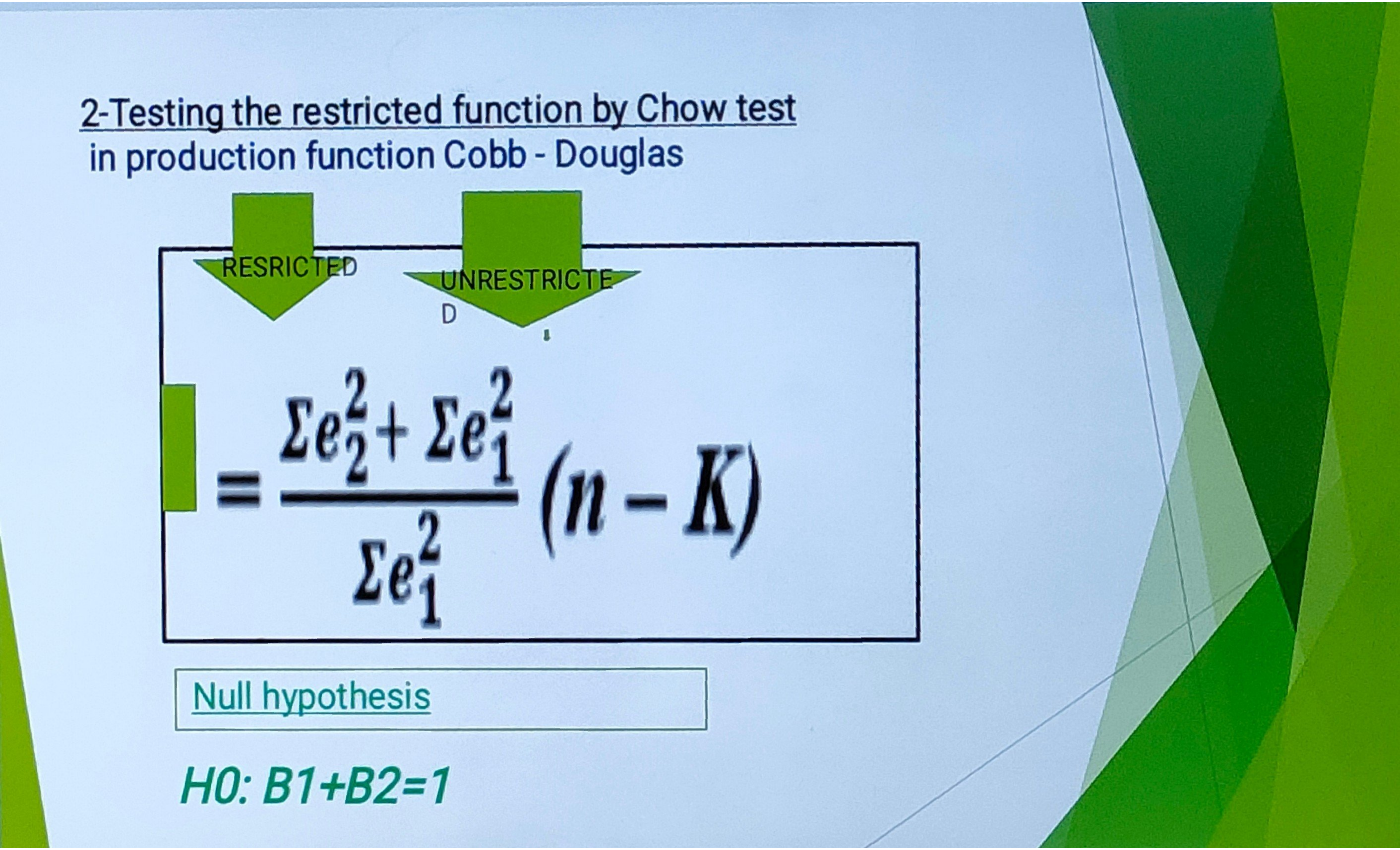
The test consists of breaking the sample into two sub-samples (to explore the structural change or the breakpoint estimating the equation for each of them, and then comparing the RSS *from* the separate equations with that of the whole sample.

Null hypothesis, reflects no significance difference between the values of the parameters in the two samples.(or no structural change or breakpoint)

Pooled whole period

Sub samples periods 1 and 2

$$= \frac{\left[\Sigma e_p^2 - \left(\Sigma e_1^2 + \Sigma e_2^2\right)\right]/K}{\left[\Sigma e_1^2 + \Sigma e_2^2\right] / \left(n_1 + n_2 - 2K\right)} =$$



2-Testing the restricted function by Chow test in production function Cobb - Douglas

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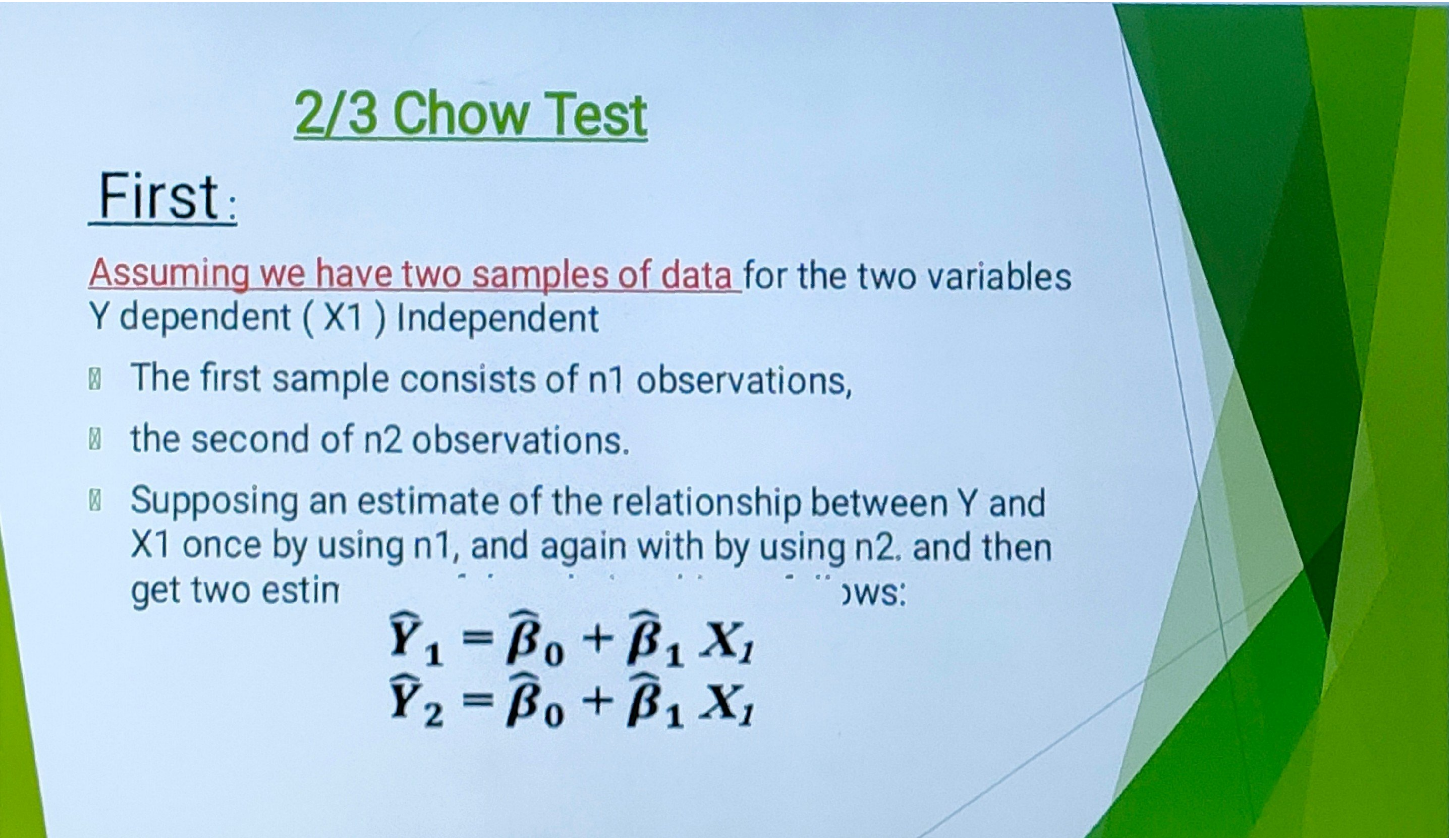
$$= \frac{\Sigma e_2^2 + \Sigma e_1^2}{\Sigma e_1^2}(n-K)$$

Null hypothesis

H0: B1+B2=1



2/3 Chow Test

First:

Assuming we have two samples of data for the two variables Y dependent (X1) Independent

- The first sample consists of n1 observations,
- the second of n2 observations.
- Supposing an estimate of the relationship between Y and X1 once by using n1, and again with by using n2. and then get two estin ows:

$$\hat{Y}_1 = \hat{\beta}_0 + \hat{\beta}_1 X_1$$
$$\hat{Y}_2 = \hat{\beta}_0 + \hat{\beta}_1 X_1$$

- Here we would like to test whether the two relationships or estimates significantly different or not. null hypothesis, reflects no significance difference between the values of the parameters in the two samples.
- **Example: Make Chow test to the following information:**
- Assuming that the estimated consumption function for the period (1980-1989) in the Egyptian economy was as follows:

$$\hat{C}_1 = 3315.27 + 0.51\,Y$$

[redacted] $\Sigma e_i^2 = 323313$

[redacted]

[redacted]

- Estimating the same function for the period (1990 - 1999) in the Egyptian economy, the result as follows:

$\hat{C}$[redacted]$_{2} = 1545.57 + 0.61\ Y$

[redacted]

Σe^{2}[redacted]$_{2} = 128552$

[redacted]

[redacted]

- To test the stability of the consumption function' parameters across the two periods, the same function should be estimated for the period as a whole years (1980-1999)(20 years) and the results as follows:

$$\hat{C} = 850.23 + 0.63\,Y$$

[redacted]

$$\Sigma e_\rho^2 = 1062082$$ [redacted]

[redacted]

[redacted]

whereas Σe_ρ^2 means the sum of the squares of the errors or residuals for the pooled period as a whole, so we put the symbol ρ as a subscript.

The Solution: the steps of chow test

- Test Chow focuses on the sum of the squares of errors for the first sample n 1 , and the second sample n 2 and for both (n 1+n 2) . Then the following should be calculated:

1- The sum of the squares of the errors or residual between the two samples is as follows:

$$(\Sigma e_1^2 + \Sigma e_2^2) = 4518654$$

With degree of freedom:

$$(n_{1+}n_2) - 2K = 20 - 4 = 16$$

2- The difference between the sum of the squares of the errors for the period as a whole and the sum of the squares of the errors of the first and second samples:

[redacted] $\Sigma e_p^2 - (\Sigma e_1^2 + \Sigma e_2^2) = 610217$

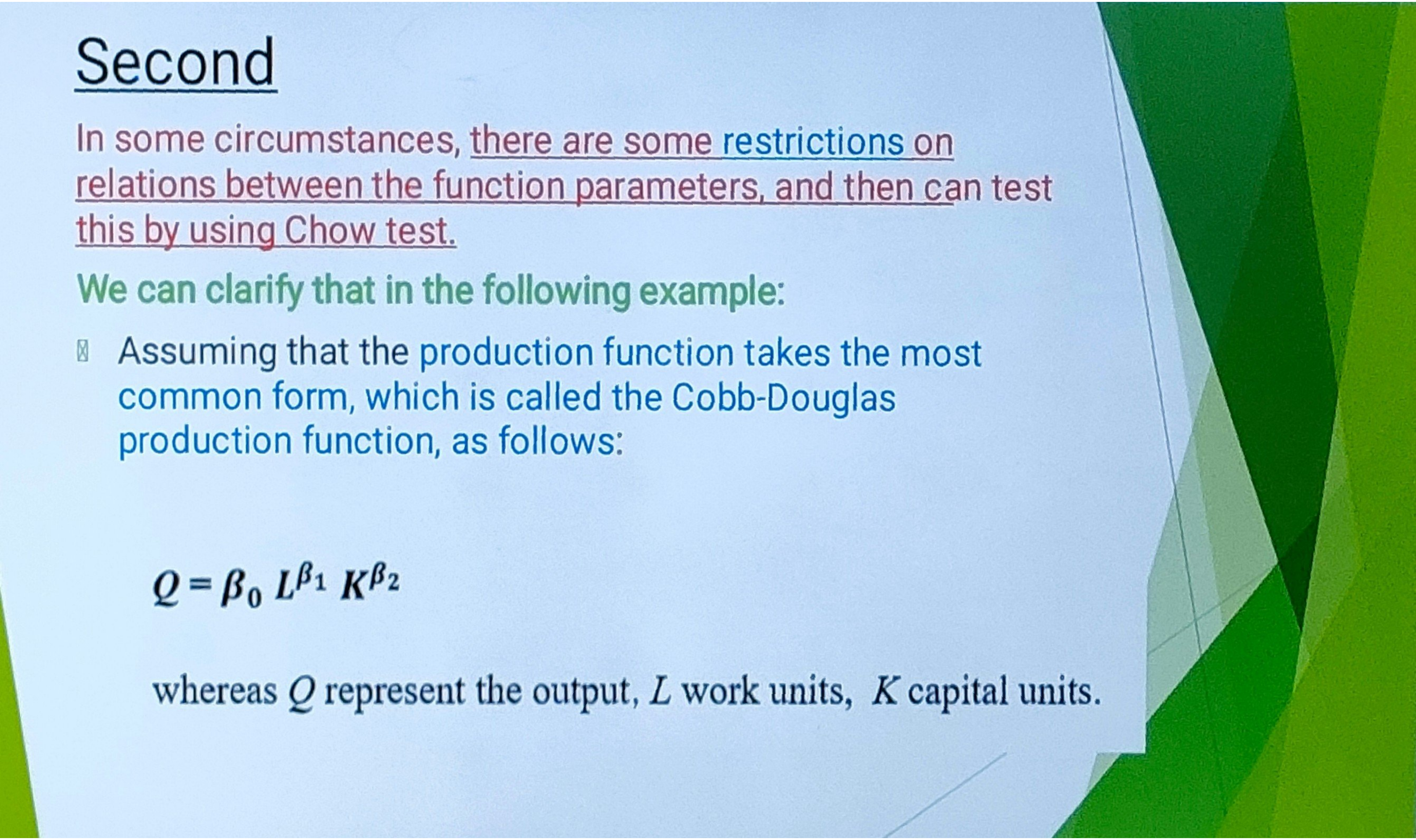
With degree of freedom:

$K =$ [redacted] $= 2$

3- Calculate F*:

$$F^* = \frac{\left[\Sigma e_p^2 - (\Sigma e_1^2 + \Sigma e_2^2)\right]/K}{\left[\Sigma e_1^2 + \Sigma e_2^2\right]/(n_1+n_2-2K)} = \text{[redacted]} = \frac{610217/2}{451865/16} = 10.8$$

By comparing Calculated F * with its tabulated at degrees of freedom 2, 16 and the level of significance of 5% (confidence level 95%) , it is clear that F *(10.8) is greater than its tabulated value (3.63) and then **reject** the null hypothesis, which reflect no significance difference between the values of the parameters in the two samples. This means that there is a significance difference in the parameter values between the two samples (a real existence of the breakpoint in the data AT YEAR 1990).



Second

In some circumstances, there are some restrictions on relations between the function parameters, and then can test this by using Chow test.

We can clarify that in the following example:

- Assuming that the production function takes the most common form, which is called the Cobb-Douglas production function, as follows:

$$Q = \beta_0 \, L^{\beta_1} \, K^{\beta_2}$$

whereas Q represent the output, L work units, K capital units.

- We chose production function Cobb - Douglas because they are subject to a fundamental limitation is that the **sum** of output elasticities with respect to labor and capital respectively is equal to one,

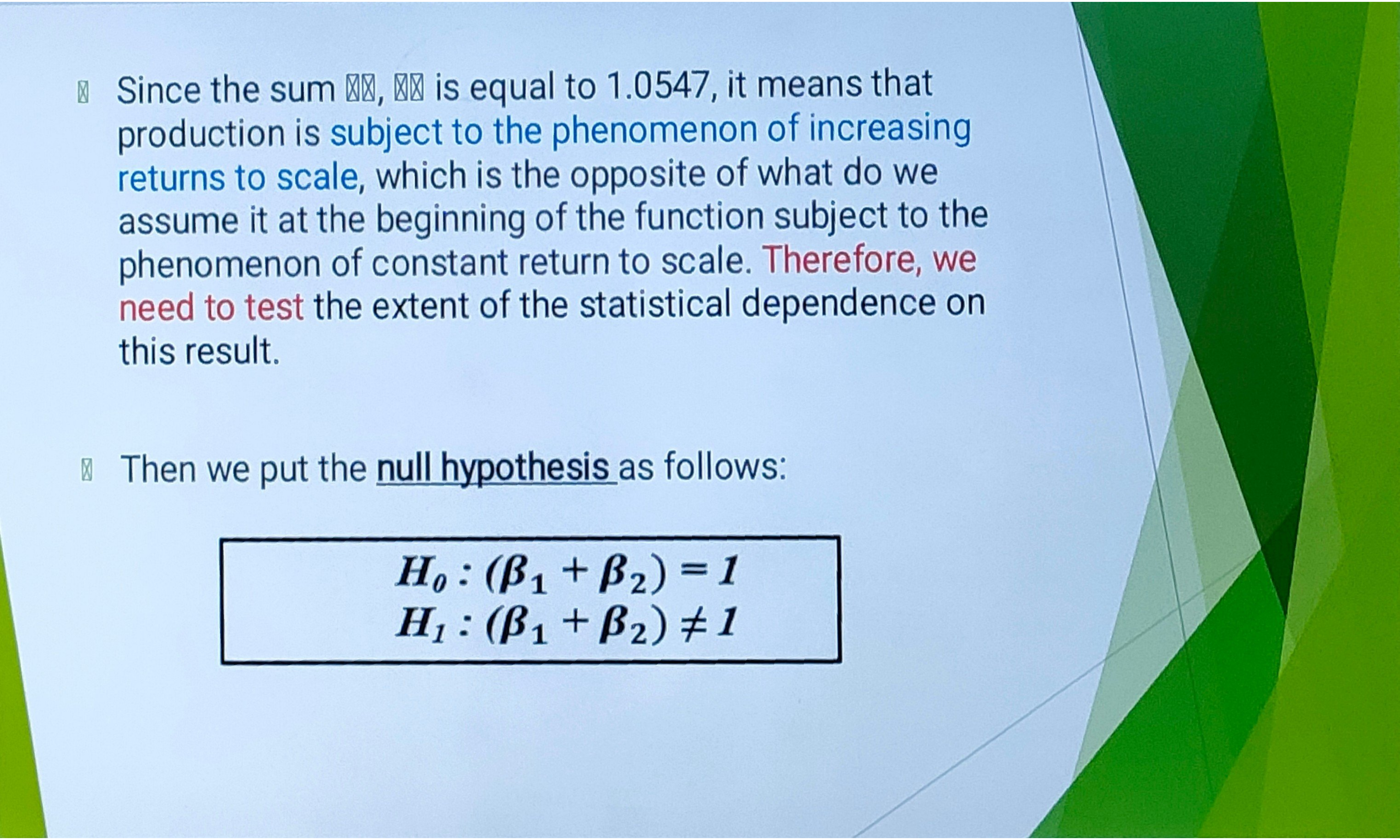
- **Or**
- which is known economically as the constant return to scale.

Example: Make Chow test to the following information:

- Assuming data is collected on each of the Q , L , K for a sample consisting of 30 observations in an industry, the results of estimating the production function with no constant return to scale's constraint as follows:

$$Q = \beta_0 L^{0.8223} K^{0.2324}$$

[redacted] $\Sigma e_1^2 = 4.64$ [redacted]



- Since the sum ⊠⊠, ⊠⊠ is equal to 1.0547, it means that production is subject to the phenomenon of increasing returns to scale, which is the opposite of what do we assume it at the beginning of the function subject to the phenomenon of constant return to scale. Therefore, we need to test the extent of the statistical dependence on this result.

- Then we put the null hypothesis as follows:

$$
\boxed{\begin{array}{c} H_0 : (\beta_1 + \beta_2) = 1 \\ H_1 : (\beta_1 + \beta_2) \neq 1 \end{array}}
$$

- This can be tested by using F* according to the following steps:

1- We perform a regression according to the constraint: $(\beta_1 + \beta_2) = 1$ By expressing β_2 to equal $(1-\beta_1)$, and substituting this into the production function as follows:

$$Q = \beta_0 \, L^{\beta_1} \, K^{1-\beta_1}$$

By estimating this function, the result of this **restricted function** can be obtained as follows:

$$Q = \widehat{\beta}_0 \, L^{0.7431} \, K^{0.2569}$$

[redacted]

$$\Sigma e^{2}_{\boxed{2}} = 6.45$$

[redacted]

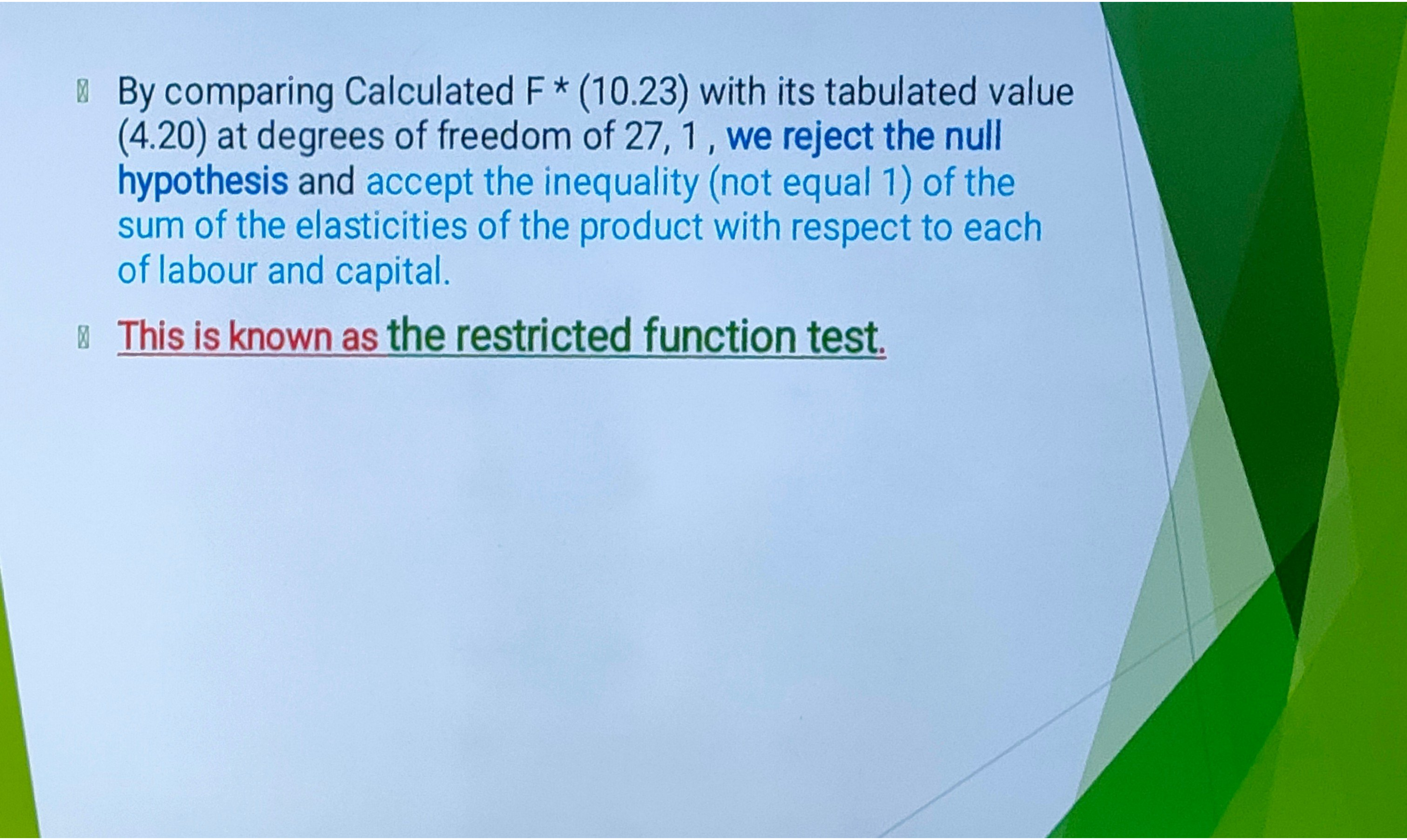
That is, we now have the sum of the error squares of the unconstrained function Σe_1^2 (Unrestricted Function), and the sum of the error squares of the restricted function Σe_2^2.

The Solution :the steps of Chow test

Tintner suggested the next test:

$$F^* = \frac{\Sigma e_2^2 + \Sigma e_1^2}{\Sigma e_1^2} (n - K)$$

$$= \frac{6.45 + 4.64}{4.64} (30 - 3) = \frac{1.81}{4.64} (27) = 10.23$$



- By comparing Calculated F * (10.23) with its tabulated value (4.20) at degrees of freedom of 27, 1 , **we reject the null hypothesis** and accept the inequality (not equal 1) of the sum of the elasticities of the product with respect to each of labour and capital.
- This is known as the restricted function test.